UNIVERSITY OF MUMBAI

SYLLABUS

Programme: M. Phil./ Ph.D.
Subject: Mathematics
(Credit Based Semester and Grading System with effect from the academic year 2013-14)

Admission Criteria and Revised syllabus in Mathematics
As per credit based system
M. Phil./ Ph.D. 2013-14

Admission Criteria:

The candidates who have passed the PET (Mathematics) conducted by University of Mumbai or have passed the NET/SET examination or have a M. Phil. (Mathematics) degree as per the UGC guidelines are eligible to appear for an Interview the Pre-Ph.D.(Mathematics) Programme. The candidates who have qualified in the Interview are be eligible for admission to the Pre- Ph. D (Mathematics) Programme. The candidates seeking admission for the M. Phil.(Mathematics) Programme shall have to appear for an Entrance Examination. The candidates who have passed the NET/SET examination are exempt from appearing for the Entrance Examination. The Entrance Examination consists of a written test for a total of 100 marks. A candidate has to score 50 or more marks to pass the Entrance Examination.

Courses and credits:

The M. Phil./ Ph.D. (Mathematics) Programme is consists of semester I & II (18 Credits). Each semester is of four months duration. There are three courses in each semester. Each course is assigned 3 credits.

Semester I: (3+3+3=9 Credits)

Paper –I (Research Methodology) (3 Credits)

The said Paper consists of:

1. Latex and Beamer
2. At least one Mathematics software Maxima/ Scilab / any other such software offered by a re search guide
3. Working knowledge of MathSciNet, JSTORE.

Paper- II (3 Credits) and Paper-III (3 Credits) are from the following:

1. Algebra-I
2. Analysis-I
3. Topology-I
4. Discrete Mathematics-I
5. Any one semester course designed and offered by a research guide/ external expert.

Semester II: (3+3+3=9 Credits)

Paper-I (Research Methodology) (3 Credits)

This Paper consists of any of the following equivalents:

1. ATM School participation
2. Review of a minimum of two research papers under the guidance of a teacher
3. A Reading Course under the guidance of a teacher.

Paper- II (3 Credits) and Paper-III (3 Credits) are from the following:
1. Algebra-II
2. Analysis-II
3. Topology-II
4. Discrete Mathematics-II
5. Any one semester course designed and offered by a research guide/ external expert.

Teaching Pattern:

There are Two lectures per week per Paper (1 lecture/period is of one hour duration)

Algebra-I (30 lectures)

Ideals, Local rings, Localization of rings and modules, Applications. Noetherian modules, Primary decomposition, Artinian modules, Length of a module.

Integral element, Integral extension, integrally closed domain, Finiteness of integral closure.

Valuation rings, Discrete valuation rings, Dedekind domains.

References
1. N. Jacobson, Basic Algebra, Vol I & II, Hindustan Publishing Corporation, New Delhi.
2. D.S. Dummit, R.M. Foote, Abstract Algebra, John Wiley & Sons, Singapore.
3. M. F. Atiyah and I. G. Macdonald, Introduction to commutative Algebra, Addison-Wesley,

Reading.
4. N. S. Gopalkrishnan, Commutative Algebra, Oxonian Press Pvt. Ltd, New Delhi.
5. S. Lang, Algebra, Addison-Wesley Publishing Company, Singapore.

Algebra II (30 lectures)

Modules, Free modules, Exact sequences, Projective modules, Injective modules, Tensor products, Flat modules.

Filtered rings and modules, Completion, I- adic filtration, Associated graded rings.

Complexes, Derived functors, Homological dimension.

References
1. N. Jacobson, Basic Algebra, Vol I & II, Hindustan Publishing Corporation, New Delhi.
2. D.S. Dummit, R.M. Foote, Abstract Algebra, John Wiley & Sons, Singapore.
3. M. F. Atiyah and I. G. Macdonald, Introduction to commutative Algebra, Addison-Wesley,

Reading.
4. N. S. Gopalkrishnan, Commutative Algebra, Oxonian Press Pvt. Ltd, New Delhi.
5. S. Lang, Algebra, Addison-Wesley Publishing Company, Singapore.

Analysis I (30 lectures)

C(X)-Spaces of continuous functions on a metric space X; the topologies of C(X) with respect to X compact, locally compact (and Hausdorff) cases; Discussion on norms, seminorms, induced topology. Discussion of C_c (X), C_o (X) etc. Discussion of Weierstrass theorem and its different proofs; Stone Weierstrass theorem, Tietze's extension theorem.

Normed linear spaces; Discussion of Finite and infinite dimensional cases. Banach spaces, closed graph theorem, open mapping theorem, Uniform boundedness principle, Banach Steinhaus theorem; Equicontinuity. Inner product spaces, Hilbert spaces, orthonormal basis, Direct Sum, Dual space of a Hilbert space. Riesz representation theorem.

Brief treatment of Lebesgue measure and Integral in general setting; Lp spaces, completeness, Duality , Reflexivity. Detailed study of L^1 (0, 2π] and L^1 (R) and its connection to Fourier Analysis. Fourier transform on L^1 ; Poisson Summation formula; Fourier inversion formula, Riemann Lebesgue Lemma, Fourier transform on L^2 ; Parseval's Identity, Plancherel theorem. The Schwartz class S of rapidly decreasing functions and its topology; Tempered distributions T and its topology; Fourier transform as a bijection on S and T.

References

1. Richard Beals, Analysis- An Introduction, Cambridge University Press.
2. E. Hewitt and K. Stromberg, Real and Abstract Analysis, Springer- Narosa.
3. J.B. Conway, A course in Functional Analysis, Springer Graduate Texts in Mathematics.
4. Walter Rudin, Functional Analysis, McGraw Hill.
5. R. Strichartz, A guide to Distribution Theory and Fourier Transforms, CRC Press.
6. Kolmogorov and Fomin, Measure, Lebesgue Integral and Hilbert Space, Academic Press.

Analysis II (30 lectures)

Detailed study of one of the following topics:

1. Operator theory on Hilbert spaces and Spectral analysis.

2. Classical Differentiation, Dini's derivatives, Functions of Bounded Variation, Absolute conti nuity, Decomposition of measures, Radon Nikodym derivative theorem and Radon Nikodym theorem; Its applications to Financial Mathematics.

3. Hilbert space techniques, orthonormal basis, theory of wavelets, multiresolution analysis, wavelet basis for L^2 .

4. Theory of Distributions, elliptic PDE.

5. Harmonic Analysis on locally compact abelian groups.

6. Brownian motion, Ito integral, Stochastic Differential equations and Application to Financial Mathematics.

7. Several Complex Variables.

8. Ergodic theory

References

1. Richard Beals, Analysis- An Introduction, Cambridge University Press.
2. E. Hewitt and K. Stromberg, Real and Abstract Analysis, Springer- Narosa.
3. J.B. Conway, A course in Functional Analysis, Springer Graduate Texts in Mathematics.
4. Walter Rudin, Functional Analysis, McGraw Hill.
5. R. Strichartz, A guide to Distribution Theory and Fourier Transforms, CRC Press.
6. Kolmogorov and Fomin, Measure, Lebesgue Integral and Hilbert Space, Academic Press.
7. Gilbert G. Walter, Wavelets and other Orthogonal Systems with Applications, CRC Press.
8. H.I. Resnikoff and R.O. Wells, Wavelet Analysis, the scalable structure of Information, Springer.
9. I.Karatzas and S.E. Shreve, Brownian Motion and Stochastic Calculus Applied to Finance, Springer.
10. Lamberton and Lapeyre, Introduction to Stochastic Calculus Applied to Finance, Chapman and Hall.

Topology I (30 lectures)

Homotopy, Fundamental Group, Homotopy Equivalence.

Covering Spaces, Classification of Covering Spaces, Covering spaces and Fundamental Group, Covering Transformations.

Classification of Surfaces, Seifert-van Kampen Theorem and applications.

References

1. James R.Munkres, Topology, Pearson Education, New Delhi.
2. John M.Lee, Introduction to topological manifolds, Springer-Verlag, new York.
3. W.S.Massey, Algebraic Topology an introduction, Harcourt Bruce& World Inc, New York.
4. Allen Hatcher, Algebraic Topology, Cambridge University Press.

Topology II (30 lectures)

Simplicial Complexes, Triangulable spaces, examples, Abstract simplicial complexes, Simplicial approximation theorem, Simplicial Homology, applications.

Singular homology, Homotopy Invariance.

Homology and the Fundamental Group, Meyer-Vietoris theorem, applications.

References

1. James R.Munkres, Topology,
2. John M.Lee, Introduction to topological manifolds,
3. W.S.Massey, Algebraic Topology an introduction,
4. Allen Hatcher, Algebraic Topology, Cambridge University Press.

Discrete Mathematics I (30 lectures)

Pigeon hole principle, Ramsey Theory, Some bounds, Addition and Multiplication principles, counting Techniques, Elementary graph Theory, Connected, Eulerian, Hamiltonian graphs, Theorems of Dirac and Posa, Ford-Fulkerson Theorem, Matching Algorithm, Hungarian Algorithm, Eigenvalues of graph, Directed graphs. In addition one of the following topics:

1.Advance graph theory
2.Algorithmic graph theory
3.Generalize Quadrangles
4.Tournaments

References

1. Biggs, Norman, Algebraic Graph Theory, Cambridge University Press.
2. Godsil, Chris; Royle, Gordon , Algebraic Graph Theory, GTM 207, Springer-Verlag.
3. Douglas B. West, Introduction to Graph Theory, Prentice Hall India.
4. William Kocay, Donald L. Kreher, Graphs, Algorithms and Optimization, Chapman and Hall.

Discrete Mathematics II (30 lectures)

Finite fields, Finite geometries, Projective planes, Affine planes, Difference sets, Designs, Construction of Design, Symmetric Design, Binary linear Codes, Generator matrix and check matrix, decoding, Spheres packing, Gilbert-Varshamov bound, single error correcting Hamming Codes. In addition one of the following topics:

1.Advance Coding Theory
2.Matroid Theory
3.Game Theory
4.Combinatorial Matrix Theory

References

1. Thomas Beth & D. Jungnickel & H. Lenz, Design Theory, Volume 1 & 2 Encyclopedia of mathematics and its applications.
2. W. D. Wallis, Further Computational and Constructive Design Theory, Kluwer Academic Publishers.
3. Peter Dembowski, Finite Geometries, Classics in Mathematics, Springer.
4. J.H. van Lint, Introduction to Coding Theory, Springer.
5. Florence Jessie MacWilliams & Neil James Alexander Sloane, The theory of Error Correcting Codes, North-Holland.
6. Richard A. Brualdi, Drago M. Cvetkovi, A Combinatorial Approach to Matrix Theory and its Applications, Chapman & Hall/CRC Press.
7. Richard A. Brualdi, Herbert John Ryser, Combinatorial Matrix Theory, Cambridge University Press.
8. Guillermo Owen, Game Theory. AcademicPress.
9. Philip D. Straffin, Game theory and Strategy, Mathematical Association of America Textbook.

Scheme of Examination

The scheme consists of seminars/assignments and two tests in each semester. Each candidate will have to submit a Dissertation which will be assessed by an external examiner.

Attendance: 75% attendance to the lectures is essential to qualify for appearing for the tests.
First Test: There will be a mid-semester written test for 40 marks on the topics covered of 2 hours duration in all the three courses.

Seminars: There will be continuous assessment in the form of Seminars/Lab works and assignments during the semester and 10 marks are be allotted for this in each course.

Final Test: The final test is for 50 marks and is of 2 hours duration in each course on the entire syllabus of the semester.

Candidates securing a total of 50 or more marks in a course from the first test, seminar/ Lab works and the final test with at least 25 marks in the final test will be declared to have passed the respective course and earned 3 credits. Candidates failed in a course will be allowed to take a supplementary examination (at most two attempts) for the final test in the respective course. If a candidate participates in a ATM work shop as course 3 in semester II, then the Head of the Department will request the Coordinator of the ATM work shop to give a grade to the candidate based on the participation.

A candidate who secures a total of **18 credits** from the six courses of Semester I & II together will be declared to have passed the Ph.D. programme/ Theory part of M. Phil programme and he/she will be allowed to submit the Ph.D./M. Phil. dissertation. There is no grade point for the Dissertation work. The Dissertation work is accepted or rejected.

The candidates selected for the Ph.D. programme, after acquiring the M. Phil. (mathematics) degree of University of Mumbai need no credits to be earned. Such candidates will be allowed to submit the Ph.D. dissertation. The candidates selected for the Ph.D. programme after acquiring the M. Phil. (Mathematics) degree of any other University will have to earn 18 credits if there is no equivalent course work prescribed for the M. Phil. programme of the respective other University.

If a candidate wants to leave the Ph.D. programme after earning 18 credits in the Ph.D. Programme, he/she may be allowed to register for the M. Phil. Programme.In such a case, the 18 credits will be transferred to his/her M. Phil. Programme and he/she will be allowed to submit the Dissertation.

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