UNIVERSITY OF MUMBAI

Syllabus for PET

Course: Mathematics

with immediate effect

University of Mumbai

Syllabus for PET Exam Mathematics

Paper 1: Research Methodology

Note: For Paper 1- 50% questions be asked on 1 to 6 and 50 % questions be asked on 7 to 13.

1. **Problem Identification & Formulation :** Definition and formulating the research problem, Necessity of defining the problem, Importance of literature review in defining a problem, Research Question – Investigation Question – Measurement Issues – Hypothesis- Qualities of a good hypothesis – Null hypothesis & Alternative Hypothesis. Hypothesis Testing – Logic & importance,

2. **Research Design:** Concept and Importance in Research – Features of good research design- Exploratory Research Design – Concept, Types and uses, Descriptive Research Design- concept, types and uses. Experimental Design- Concept of Independent & dependent variables.

3. Data Bases: Use of Encyclopedias, Research Guides, Handbook etc., Academic databases like Mathscinet, Scopus, Web of Science etc.

4. Interpretation of Data and Paper Writing: Graphical interpretation of data, Layout of a Research Paper, Journals, Ethical issues related to publishing, Plagiarism and Self- Plagiarism.

5. Reasoning and Mental ability: Analogy, Logical reasoning and aptitude, Classification, Series, Coding-Decoding, Direction Sense, Representation Through Venn Diagrams, Mathematical Operations, Arithmetical Reasoning, Inserting the Missing Character, Numbers and Indexing, Ranking and Time Sequence Test, Eligibility Test, Comprehension questions, Statement & assumptions, Statement & conclusion, Deductive arguments, Russell's paradox.

6. IPR : Types, Copyrights in Scientific work, Patents in scientific research, Writing apatent specification, patent filing and grant, infringement. Gene patenting, Farmer's rights, Plant Breeder's rights, Traditional knowledge and protection.

7. Solutions - Algebraic equations - Simultaneous linear Equations – Trigonometric/ Exponential functions, Matrix Algebra & Determinants.

8.**Fundamentals of Mathematics :** (a) Set, Operations on sets and functions, (b) Pigeon-hole principle, Well-ordering principle and principle of mathematical induction, (c) Number Systems, Fundamental Theorem of Arithmetic, GCD & LCM of numbers and polynomials, Modular Arithmetic.

9. **Basic Concepts of Real and Complex Analysis** : Sequences and series, Continuity, Uniform continuity, Differentiabilty, Mean Value Theorem, Sequences and series of functions, Uniform convergence, Riemann integral - definition and simple

properties. Algebra of complex numbers, Analytic functions, Cauchy's Theorem and integral formula, Power series, Taylor's and Laurent's series, Residues, Contour integration.

10. Basic Concepts of Linear Algebra : Space of n-vectors, Linear dependence, Basis, Linear transformation, Algebra of matrices, Rank of a matrix, Determinants, Linear equations, Quadratic forms, Characteristic roots and vectors.

11. Basic Concepts of Probability : Sample space, Discrete probability, Simple theorems on probability, Independence of events, Bayes Theorem, Discrete and continuous random variables, Binomial, Poisson and Normal distributions; Expectation and moments, Independence of random variables, Chebyshev's inequality.

12. Basic Concepts of Algebra : Group, subgroups, Normal subgroups, Quotient Groups, Homomorphisms, Cyclic Groups, Permutation Groups, Cayley's Theorm, Rings, Ideals, Integral Domains, Fields, Polynomial Rings.

13**. Basic concepts of Differential Equations :** First order ODE, singular solutions initial value Problems of First Order ODE, General theory of homogeneous and non-homogeneous Linear ODE, Variation of Parameters. Lagrange's and Charpit's methods of solving First order Partial Differental Equations. PDE's of higher order with constant coefficients.

References:

1. As prescribed for the Mumbai University M. Sc. Mathematics syllabus.

2. Kothari C. R., 1990. Research Methodology: Methods and Techniques New AgeInternational 418p.

3. Research Methodology and Scientific Writings- C George Thomas

Paper 2: Subject Specific-Mathematics.

1. Real Analysis : Riemann integrable functions; improper integrals, their convergence and uniform convergence. Eulidean space R", Bolzano-Weierstrass theorem, compact Subsets of R", Heine-Borel Theorem, Fourier series. Continuity of functions on R", Differentiability of F from R" to R^m . Properties of differential, partial and directional derivatives, continuously differentiable functions. Taylor's series. Inverse function theorem, Impliecit function theorem.

2. Complex Analysis : Cauchy's theorem for convex regions. Power series representation of Analytic functions. Liouville's theorem, Fundamental theorem of algebra, Riemann's theorem on removable singularities, maximum modulus principle. Schwarz Iemma, Open Mapping theorem, Casorattl-Weierstrass-theorem, Weierstrass's theorem on uniform convergence on compact sets, Bilinear transformations,

3. Algebra : Symmetric groups, Alternating groups, Simple groups, Rings, Maximal Ideals, Prime Ideals, Integral domains Euclidean domains, principal Ideal domains,

Unique Factorisation domains, quotient fileds, Finite fields, Algebra of Linear Transformations, Reduction of matrices to Canonical Forms, Inner Product Spaces, Orthogonality,

4. Advanced Analysis : Elements of Metric Spaces, Convergence, continuity, compactness, Connectedness, Weierstrass's approximation Theorem, Completeness, Bare category theorem, Labesgue measure, Labesgue Integral, Differentiation and Integration.

5. Advanced Algebra : Conjugate elements and class equations of finite groups, Sylow theorems, solvable groups, Jordan Holder Theorem, Direct Products, Structure Theorem for finite abelian groups, Characteristic of Field, Field extensions, Elements of Galois theory, solvability by Radicals , Ruler and compass construction.

6. Functional Analysis : Banach Spaces, Hahn-Banach Theorem, Open mapping and closed Graph Theorems. Principal of Uniform boundedness, Boundedness and continuity of Linear Transformations, Dual Space, Embedding in the second dual, Hilbert Spaces, Projections. Orthonormal Basis, Riesz-representation theorem, Bessel's Inequality, Parsaval's identity, self adjoined operators, Normal Operators.

7. Topology : Elements of Topological Spaces, Continuity, convergence, Homeomorphism, Compactness, Connectedness, Separation Axioms, First and Second Countability, Separability, Subspaces, Product Spaces, quotient spaces. Tychonoft's Theorem, Urysohn's Metrization theorem, Homotopy and Fundamental Group.

8. Discrete Mathematics : Partially ordered sets, Lattices, Cornplete Lattices, Distrbutive lattices, Complements, Boolean Algebra, Boolean Expressions, Application to switching circuits, Elements of Graph Theory, Eulerian and Hamiltonian graphs, planar Graphs, Directed Graphs, Trees, Permutations and Combinations, Pigeonhole principle, principle of Inclusion and Exclusion, Derangements.

9. Ordinary and partial Differential Equations : Lipchitz conditions, Existence and Uniqueness theorem for the solution of n^{th} ODE. Green's function, sturm Liouville Boundary Value Problems,

10. Partial Differential Equations: Cauchy Problems and Characteristics Equations, Classification of Second Order PDE, Separation of Variables for heat equation, wave equation and Laplace equation, Special functions.

11. Differetial Geometry : Plane curves, Space curves-their curvature and torsion; Serret Frehat Formula; Fundamental theorem of plane and space curves; Curves on sirfaces, First and second fundamental form; Gaussian curvatures; Principal directions and principal curvatures; Goedesics, Fundamental equations of surface theory.

12. Numerical analysis : Finite differences, Interpolation; Numerical solution of algebric equation; Iteration; Newton-Raphason Method; Solution on Linear system; Direct method; Gauss elminaiton method; Matrix-Inversion eigenvalue problems; Numerical differentiation and integration. Numerical solution of ordinary differential

equation; iteration method, Picard's method , Euler's method and Modified Euler's method.

13. Integral Transform : The Laplace transform, The Fourier Transform, The Mellin Transform, The Z-Transform .

14. Combinatorics - Counting Principles, Generating Functions, Recursion, Boolean Algebra, Distributive Lattices, Graphs, Trees, Connectivity and Coloring of Graphs

15. Probability Introduction to probability, probability space, events. Classical probability spaces: uniform probability measure, fields, finite fields, finitely additive probability, Inclusion-exclusion principle, σ-fields, σ-fields generated by a family of sets, σ-field of Borel sets, Limit superior and limit inferior for a sequence of events. Probability measure, Continuity of probabilities, First Borel-Cantelli lemma, Discussion of Lebesgue measure on σ-field of Borel subsets of assuming its existence, Discussion of Lebesgue integral for non-negative Borel functions assuming its construction. Discrete and absolutely continuous probability measures, conditional probability, total probability formula, Bayes formula, Independent events. Random variables, simple random variables, discrete and absolutely continuous random variables, distribution of a random variable, distribution function of a random variable, Bernoulli, Binomial, Poisson and Normal distributions, Independent random variables, Expectation and variance of random variables both discrete and absolutely continuous. Conditional expectations and their properties, characteristic functions, examples, Higher moments examples, Chebyshev inequality, Weak law of large numbers, Convergence of random variables, Kolmogorov strong law of large numbers, Central limit theorem .

16. Statistical Methods and Data Analysis : Tests for mean and variance in the normal distribution : one-population and two-population cases; related confidence intervals. Tests for product moment, partial and multiple correlation coefficients; comparison of k linear regressions. Fitting polynomial regression; related test Analysis of discrete date: chi-square test of goodness of fit, contingency tables. Analysis of variance : one-way and two-way classification (equal number of observations per cell). Large sample tests through normal approximation. Nonparametric tests : sign test, Median test, Mann-Whitney test, Wilcoxon test for one and two-samples, rank correlation and test of independence.

References: :As prescribed for the Mumbai University M. Sc. Mathematics syllabus.

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