UNIVERSITY OF MUMBAI

Syllabus

for M. Sc./ M. A. Semester I & II (CBCS)
Program: M. Sc./ M. A.

Course: Mathematics

with effect from the academic year 2020-2021

Syllabus M.Sc./M.A. Part I (Sem. I & II)

Choice Based Credit System (CBCS)

Sr. No.	Subject code	Units	Subject	Credits	L/W
Semester I					
Algebra I					
01	PSMT/PAMT 101	Unit I	Dual Spaces	05	04
		Unit II	Determinants and Characteristics Polynomial		
		Unit III	Triangulation of Matrices		
		Unit IV	Bilinear Forms		
Analysis I					
02	PSMT/PAMT 102	Unit I	Euclidean Space	05	04
		Unit II	Differentiable Functions		
		Unit III	Inverse Function Theorem and Implicit Function Theorem		
		Unit IV	Riemann Integration		
Complex Analysis					
03	PSMT/PAMT 103	Unit I	Holomorphic Functions	05	04
		Unit II	Contour Integration and Cauchy-Goursat theorem		
		Unit III	Holomorphic Functions and Their Properties		
		Unit IV	Residue Calculus and Mobius Transformation		
Ordinary Differential Equations					
04	PSMT/PAMT 104	Unit I	Existence and Uniquness of Solutions	05	04
		Unit II	Linear Equations with Constant Coefficients		
		Unit III	Linear Equations with		

			Variable Coefficients		
		Unit IV	Strum_Liouville Problem and Qualitative Properties of Solutions		
Discrete Mathematics					
05	PSMT/PAMT 105	Unit I	Number Theory	04	04
		Unit II	Advanced Counting		
		Unit III	Recurrence Relations		
		Unit IV	Polyas Theory of Counting		
Semester II					
Algebra II					
01	PSMT/PAMT 201	Unit I	Groups and Group Homomorphisms	05	04
		Unit II	Group Acting on Sets and Sylow Theorems		
		Unit III	Rings and Fields		
		Unit IV	Divisibility in Integral Domains		
Topology					
02	PSMT/PAMT 202	Unit I	Topology and Topological Spaces	05	04
		Unit II	Connected Topological Spaces		
		Unit III	Compact Topological Spaces		
		Unit IV	Metrizable Spaces and Tychonoff Theorem		
Analysis II					
03	PSMT/PAMT 203	Unit I	Measures and Measurable Sets	05	04
		Unit II	Measurable functions and their Integration		
		Unit III	Convergence Theorems on Measure space		
		Unit IV	Space of Integrable functions		

Partial Differential Equations					
04	PSMT/PAMT 204	Unit I	First Order Partial Differential Equations	05	04
		Unit II	Second Order Partial Differential Equations		
		Unit III	Green's Functions and Integral Representations		
		Unit IV	The Diffusion Equation and Parabolic Differential Equations		
Probability Theory					
05	PSMT/PAMT 205	Unit I	Probability Basics	04	04
		Unit II	Probability Measure		
		Unit III	Random Variables		
		Unit IV	Limit Theorems		

Teaching Pattern for Semester I and II
1. Four lectures per week per course. Each lecture is of 60 minutes duration.
2. In addition, there shall be tutorials, seminars as necessary for each of the five courses.

Semester-I

PSMT101 /PAMT101 ALGEBRA I

Course Outcomes:

1. Students will be able to understand the notion of dual space and double dual, Annihilator of a subspace and its application to counting the dimension of a finite dimensional vector space, Basics of determinants, applications to solving system of equations, Nilpotent operators, invariant subspaces and its applications, Bilinear forms and spectral theorem with examples of spectral resolution and Symmetric bilinear form and Sylvesters law.

2. Students will be able to understand the applicability of the above concepts in different courses of pure and applied mathematics and hence in other disciplins of science and technology.

Unit I. Dual spaces (15 Lectures)
Para 1 and 2 of Unit I are to be reviewed without proof (no question be asked).

1. Vector spaces over a field, linear independence, basis for finite dimensional and infinite dimensional vector spaces and dimension.

2. Kernel and image, rank and nullity of a linear transformation, rank-nullity theorem (for finite dimensional vector spaces), relationship of linear transformations with matrices, invertible linear transformations. The following are equivalent for a linear map $T : V \to V$ of a finite dimensional vector space V :

   (a) T is an isomorphism.
   (b) $\ker T = \{0\}$.
   (c) $Im(T) = V$.

3. Linear functionals, dual spaces of a vector space, dual basis (for finite dimensional vector spaces), annihilator W^o in the dual space V^* of a subspace W of a vector space V and dimension formula, a k-dimensional subspace of an n-dimensional vector space is intersection of $n-k$ many hyperspaces. Double dual V^{**} of a Vector space V and canonical embedding of V into V^{**}. V^{**} is isomorphic to V when V is of finite dimension. (ref:[1] Hoffman K. and Kunze R.).

4. Transpose T^t of a linear transformation T. For finite dimensional vector spaces: rank (T^t) =rank T, range(T^t) is the annihilator of kernel (T), matrix representing T^t. (ref:[1] Hoffman K and Kunze R)

Unit II. Determinants & Characteristic Polynomial (15 Lectures)
Rank of a matrix. Matrix of a linear transformation, change of basis, similar matrices. Determinants as alternating n-forms, existence and uniqueness, Laplace expansion of determinant, determinants of products and transposes, adjoint of a matrics. determinants and invertible linear transformations, determinant of a linear transformation. Solution of system of linear equations using Cramer's rule. Eigen values and Eigen vectors of a linear transformation, Annihilating polynomial, Characteristic polynomial, minimal polynomial, Cayley-Hamilton theorem. (Reference for Unit II: [1] Hoffman K and Kunze R, Linear Algebra).

Unit III. Triangulation of matrices (15 Lectures)
Triangulable and diagonalizable linear operators, invariant subspaces and simple matrix representation (for finite dimension). (ref: [5] N.S. Gopalkrishnan & [3] Serge Lang) Nilpotent linear transformations on finite dimensional vector spaces, index of a Nilpotent linear transformation. Linear independence of $\{u, N, \ldots, N^{k-1}u\}$ where N is a nilpotent linear transformation of index $k \geq 2$ of a vector space V and $u \in V$ with $Nu \neq 0$. (Ref: [2] I.N.Herstein).

For a nilpotent linear transformation N of a finite dimensional vector space V and for any subspace W of V which is invariant under N, there exists a subspace V_1 of V such that $V = W \oplus V_1$. (Ref:[2] I.N.Herstein).

Computations of Minimum polynomials and Jordan Canonical Forms for 3×3-matrices through examples. (Ref:[6] Morris W. Hirsch and Stephen Smale).

Unit IV. Bilinear forms (15 Lectures)
Para 1 of Unit IV is to be reviewed without proof (no question be asked).

1. Inner product spaces, orthonormal basis, Gram-Schmidt process.
2. Adjoint of a linear operator on an inner product space, unitary operators, self adjoint operators, normal operators. (ref:[1] Hoffman K and Kunze R). Spectral theorem for a normal operator on a finite dimensional complex inner product space. (ref:[4] Michael Artin, Ch. 8). Spectral resolution (examples only). (ref:[1] Hoffman K and Kunze R, sec 9.5).
3. Bilinear form, rank of a bilinear form, non-degenerate bilinear form and equivalent statements. (ref:[1] Hoffman K and Kunze R).
4. Symmetric bilinear forms, orthogonal basis and Sylvester's Law, signature of a Symmetric bilinear form. (ref:[4] Michael Artin).

Recommended Text Books

1. Hoffman K and Kunze R: Linear Algebra, Prentice-Hall India.
2. I.N.Herstein: Topics in Algebra, Wiley-India.

3. Serge Lang: Linear Algebra, Springer-Verlag Undergraduate Text in Mathematics.

4. Michael Artin: Algebra, Prentice-Hall India.

5. N.S. Gopalkrishnan: University Algebra, New Age International, third edition, 2015.

6. Morris W. Hirsch and Stephen Smale, Differential Equations, Dynamical Systems, Linear Algebra, Elsevier.

PSMT102 / PAMT102 ANALYSIS I

Course Outcomes

1. This course is the foundation course of mathematics, especially mathematical analysis.
2. Student will be able to grasp approximation of a differentiable function localized at a point.
3. Inverse function theorem helps to achieve homeomorphism locally at a point whereas implicit function theorem justifies the graph of certain functions. Indirectly or directly Unit III talks about value of a function in the neighbourhood of a known element.
4. In Unit IV, student will be able to understand the concept of Riemann integration.

Unit I. Euclidean space $\mathbb{R}^n$ (15 Lectures)
Euclidean space $\mathbb{R}^n$: inner product $\langle x, y\rangle = \sum_{j=1}^{n} x_j y_j$ of $x = (x_1, .., x_n), y = (y_1, .., y_n) \in R^n$ and properties, norm $||x|| = \sqrt{\sum_{j=1}^{n} x_j^2}$ of $x = (x_1, .., x_n) \in R^n$, Cauchy-Schwarz inequality, properties of the norm function $||x||$ on $\mathbb{R}^n$. (Ref. W. Rudin or M. Spivak).

Standard topology on $\mathbb{R}^n$: open subsets of $\mathbb{R}^n$, closed subsets of $\mathbb{R}^n$, interior A^o and boundary ∂A of a subset A of $\mathbb{R}^n$. (ref. M. Spivak)

Operator norm $||T||$ of a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ ($||T|| = sup\{||T(v)|| : v \in \mathbb{R}^n \& ||v|| \leq 1\}$) and its properties such as: For all linear maps $S, T : \mathbb{R}^n \to \mathbb{R}^m$ and $R : \mathbb{R}^m \to \mathbb{R}^k$

1. $||S + T|| \leq ||S|| + ||T||$,
2. $||RoS|| \leq ||R||||S||$,and
3. $||cT|| = |c|||T||(c \in \mathbb{R})$.

(Ref. C. C. Pugh or A. Browder)

Compactness: Open cover of a subset of $\mathbb{R}^n$, Compact subsets of $\mathbb{R}^n$ (A subset K of $\mathbb{R}^n$ is compact if every open cover of K contains a finite subover), Heine-Borel theorem (statement only), the Cartesian product of two compact subsets of $\mathbb{R}^n$ is compact (statement only),every closed and bounded subset of $\mathbb{R}^n$ is compact. Bolzano-Weierstrass theorem: Any bounded sequence in $\mathbb{R}^n$ has a converging subsequence.

Brief review of following three topics:

1. Functions and Continuity Notation: $A \subset \mathbb{R}^n$ arbitary non-empty set. A function $f : A \to \mathbb{R}^m$ and its component functions, continuity of a function($\in, \delta$ definition). A function $f : A \to \mathbb{R}^m$ is continuous if and only if for every open subset $V \subset \mathbb{R}^m$ there is an open subset U of $\mathbb{R}^n$ such that $f^{-1}(V) = A \cap U$.

2. Continuity and compactness:Let $K \subset \mathbb{R}^n$ be a compact subset and $f : K \to \mathbb{R}^m$ be any continuous function. Then f is uniformly continuous, and $f(K)$ is a compact subset of $\mathbb{R}^m$.

3. Continuity and connectedness:Connected subsets of $\mathbb{R}$ are intervals. If $f : E \to \mathbb{R}$ is continuous where $E \subset \mathbb{R}^n$ and E is connected, then $f(E) \subset \mathbb{R}$ is connected.

Unit II. Differentiable functions (15 Lectures)
Differentiable functions on $\mathbb{R}^n$, the total derivative $(Df)_p$ of a differentiable function $f : U \to \mathbb{R}^m$ at $p \in U$ where U is open in $\mathbb{R}^n$,uniqueness of total derivative, differentiability implies continuity.(ref:[1] C.C.Pugh or[2] A.Browder)

Chain rule. Applications of chain rule such as:

1. Let γ be a differentiable curve in an open subset U of $\mathbb{R}^n$. Let $f : U \to \mathbb{R}$ be a differentiable function and let $g(t) = f(\gamma(t))$. Then $g'(t) = \langle(\nabla f)(\gamma(t)), \gamma'(t)\rangle$.
2. Computation of total derivatives of real valued functions such as
   (a) the determinant function $\det(X)$, $X \in M_n(\mathbb{R})$,
   (b) the Euclidean inner product function $\langle x, y\rangle$, $(x, y) \in \mathbb{R}^n \times \mathbb{R}^n$.

(ref. M. Spivak, W. Rudin)

Results on total derivative:

1. If $f : \mathbb{R}^n \to \mathbb{R}^m$ is a constant function,then $(Df)_p = 0\ \forall p \in \mathbb{R}^n$.
2. If $f : \mathbb{R}^n \to \mathbb{R}^m$ is a linear map, then $(Df)_p = f\ \forall p \in \mathbb{R}^n$.
3. A function $f = (f_1, f_2, ..f_m) : \mathbb{R}^n \to \mathbb{R}^m$ is differentiable at $p \in \mathbb{R}^n$ if and only if each f_j is differentiable at $p \in \mathbb{R}^n$, and $(Df)_p = ((Df_1)_p, (Df_2), p, .., (Df_m)_p)$. (ref. M. Spivak).

Partial derivatives, directional derivative $(D_u f)(p)$ of a function f at p in the direction of the unit vector, Jacobian matrix, Jacobian determinant. Results such as :

1. If the total derivative of a map $f = (f_1, .., f_m) : U \to \mathbb{R}^m$($U$ open subset of $\mathbb{R}^n$) exists at $p \in U$, then all the partial derivatives $\frac{\partial f_i}{\partial x_j}$ exists at p.
2. If all the partial derivatives $\frac{\partial f_i}{\partial x_j}$ of a map $f = (f_1, .., f_m) : U \to \mathbb{R}^m$($U$ open subset of $\mathbb{R}^n$) exist and are continuous on U, then f is differentiable.(ref. W. Rudin)

Derivatives of higher order, C^k-functions, C^∞-functions.(ref. T. Apostol)

Unit III. Inverse function theorem and Implicit function theorem (15 Lectures)
Theorem (Mean Value Inequality): Suppose $f : U \to \mathbb{R}^m$ is differentiable on an open subset U of $\mathbb{R}^n$ and there is a real number such that $||f(Df)_x|| \leq M \; \forall x \in U$. If the segment $[p, q]$ is contained in U, then $||f(q) - f(p)|| \leq M||q - p||$. (ref. C. C. Pugh or A. Browder).

Mean Value Theorem: Let $f : U \to \mathbb{R}^m$ is a differentiable on an open subset U of $\mathbb{R}^n$. Let $p, q \in U$ such that the segment $[p, q]$ is contained in U. Then for every vector $\mathbf{v} \in \mathbb{R}^n$ there is a point $x \in [p, q]$ such that $\langle \mathbf{v}, f(q) - f(p) \rangle = \langle \mathbf{v}, (Df)_x(q - p) \rangle$. (ref:T. Apostol) If $f : U \to \mathbb{R}^m$ is differentiable on a connected open subset U of R^n and $(Df)_x = 0 \; \forall x \in U$, then f is a constant map.

Taylor expansion for a real valued C^m-function defined on an open subset of R^n, stationary points(critical points), maxima, minima, saddle points, second derivative test for extrema at a stationary point of a real valued C^2-function defined on an open subset of $\mathbb{R}^n$. Lagrange's method of undetermined multipliers.
(ref. T. Apostol)

Contraction mapping theorem. Inverse function theorem, Implicit function theorem.(ref. A. Browder)

Unit IV. Riemann Integration(15 Lectures)
Riemann Integration over a rectangle in $\mathbb{R}^n$, Riemann Integrable functions, Continuous functions are Riemann integrable, Measure zero sets, Lebesgues Theorem(statement only), Fubini's Theorem and applications. (Reference for Unit IV: M. Spivak, Calculus on Manifolds).

Recommended Text Books

1. C. C. Pugh, Mathematical Analysis, Springer UTM.
2. A. Browder, Mathematical Analysis an Introduction, Springer.
3. T. Apostol, Mathematical Analysis, Narosa.
4. W. Rudin, Principals of Mathematical Analysis, McGraw-Hill India.
5. M. Spivak, Calculus on Manifolds, Harper-Collins Publishers.

PSMT103 / PAMT103 COMPLEX ANALYSIS

Course Outcomes

1. In this course the students will learn about series of functions and power series. The concept of radius of convergence will be introduced and calculated.
2. This course gives insight of complex integration which is different from integration of real valued functions. In particular, Cauchy integral formula will be proved.
3. The students will learn that if a function is once (complex) differentiable then it is infinitely many times differentiable. This will be a sharp contrast with the theorems of real analysis.
4. The various properties of Möbius transformations that have a wide variety of applications along with major theorems of theoretical interest like Cauchy-Goursat theorem, Morera's theorem, Rouché's theorem and Casorati-Weierstrass theorem will be studied.

Unit I. Holomorphic functions (15 Lectures)
Note: A complex differentiable function defined on an open subset of $\mathbb{C}$ is called a holomorphic function.

Review: Complex numbers, Geometry of the complex plane, Weierstrass's M-test and its aplication to uniform convergence, Ratio and root test for convergence of series of complex numbers. (no questions to be asked).

Stereographic projection, Sequence and series of complex numbers, Sequence and series of functions in $\mathbb{C}$, Complex differential functions, Chain rule for holomorphic function.

Power series of complex numbers, Radius of convergence of power series, Cauchy-Hadamard formula for radius of convergence of power series. Abel's theorem: let $\sum_{n\geq 0} a_n(z-z_0)^n$ be a power series of radius of convergence $R > 0$. Then the function $f(z)$ defined by $f(z) = \sum a_n(z-z_0)^n$ is holomorphic on the open ball $|z-z_0| < R$ and $f'(z) = \sum_{n\geq 1} na_n(z-z_0)^{n-1}$ for all $|z - z_0| < R$. Trigonometric functions, Applications of Abel's theorem to trigonometric functions.

Applications of the chain rule to define the logarithm as the inverse of exponential, branches of logarithm, principle branch $l(z)$ of the logarithm and its derivative on $\mathbb{C} \setminus \{z \in \mathbb{C} \mid \mathrm{Re}(z) \leq 0, \mathrm{Im}(z) = 0\}$.

Unit II. Contour integration, Cauchy-Goursat theorem (15 Lectures)
Contour integration, Cauchy-Goursat Theorem for a rectangular region or a triangular region. Cauchy's theorem(general domain), Cauchy integral formula, Cauchy's estimates, The index(winding number) of a closed curve, Primitives. Existence of primitives, Morera's theorem. Power series representation of holomorphic function (Taylor's theorem).

Unit III. Properties of Holomorphic functions (15 Lectures)
Entire functions, Liouville's theorem. Fundamental theorem of algebra. Zeros of holomorphic functions, Identity theorem. Counting zeros; Open Mapping Theorem, Maximum modulus theorem, Schwarz's lemma. Automorphisms of unit disc.

Isolated singularities: removable singularities and Removable singularity theorem, poles and essential singularities. Laurent Series development. Casorati-Weierstrass's theorem.

Unit IV. Residue calculus and Mobius transformation (15 Lectures)
Residue Theorem and evaluation of standard types of integrals by the residue calculus method. Argument principle. Rouché's theorem. Conformal mapping, Mobius Transformation.

Recommended Text Books

1. J.B. Conway, Functions of one Complex variable, Springer.
2. A.R. Shastri: An introduction to complex analysis, Macmillan.
3. Serge Lang: Complex Analysis. Springer.
4. L.V. Ahlfors:Complex analysis, McGraw Hill.
5. R. Remmert: Theory of complex functions, Springer.
6. J.W. Brown and R.V. Churchill:Complex variables and Applications, McGraw-Hill.

PSMT104 / PAMT104 ORDINARY DIFFERENTIAL EQUATIONS

Course Outcomes

1. Through this course students are expected to understand the basic concepts of existence and uniqueness of solutions of Ordinary Differential Equations (ODEs).

2. In case of nonlinear ODEs, students will learn how to construct the sequence of approximate solutions converges to the exact solution if exact solution is not possible.

3. Students will be able to understand the qualitative features of solutions.

4. Students will be able to identify Sturm Liouville problems and to understand the special functions like Legendre's polynomials and Bessel's function.

5. Students will be to understand the applicability of the above concepts in different disciplins of Techonolgy.

Unit I. Existeence and Uniqueness of Solutions (15 Lectures)
Existence and Uniqueness of solutions to initial value problem of rst order ODE- both autonomous, non autonomous, ϵ-approximate solutions, Ascoli lemma, Cauchy-Peano existence theorem, Lipschitz condition, Picard's method of successive approximations, Picard-Lindelof theorem, System of Differential equations. Reduction of n-th order differential equations.
[Reference Unit-I of Theory of Ordinary Differential Equations; Earl A. Coddington and Norman Levinson, Tata McGraw Hill, India.]

Unit II. Linear Equations with constant coefficients (15 Lectures)
The second order homogeneous equations, Initial value problem for second order equations, Uniqueness theorem, linear dependance and independence of solutions, Wronskian,a formula for the Wronskian, The second order non-homogeneous equations, The homogeneous equations of order n, Initial value problem for n^{th} order equations, The non-homogeneous equations of order n, Algebra of constant coefficient operators.
[Reference Unit-II of Earl A. Coddington, An Introduction to Ordinary Differential Equations, Prentice-Hall of India.]

Unit III. Linear Equations with variable coefficients (15 Lectures)
Initial value problem for the homogeneous equation of order n,, Existence and Uniqueness theorem, solution of the homogeneous equations, Wronskian and linear independance, reduction of the order of a homogeneous equation, the non-homogeneous equations of order n,.
[Reference III of Earl A. Coddington, An Introduction to Ordinary Differential Equations, Prentice-Hall of India.]

Unit IV. Sturm-Liouville Problem & Qualitative Properties of Solutions (15 Lectures)

Eigenvalue problem, Eigenvalues and Eigenfunctions, the vibrating string problem, Sturm Liouville problems, homogeneous and non-homogeneous boundary conditions, orthogonality property of eigenfunctions, Existence of Eigenvalues and Eigenfunctions, Sturm Seperation theorem, Sturm comparison theorem. Power series solution of second order linear equations, ordinary points, singular points, regular singular points, existence of solution of homogeneous second order linear equation, solution of Legendre's equation, Legendre's polynomials, Rodrigues' formula, orthogonality conditions, Bessel differential equation, Bessel functions, Properties of Bessel function, orthogonality of Bessel functions.
[Reference Unit IV (24, 25), Unit V (Review), Unit-VII (40, 43, Appendix A) and Unit VIII (44, 45, 46, 47) : G. F. Simmons, Differential Equations with Applications and Historical Notes, Second Edition, Tata McGraw Hill, India]

Recommended Text Books

1. Earl A., Coddington and Norman Levinson, Theory of Ordinary Differential Equations; Tata McGraw Hill, India.

2. Earl A. Coddington, An Introduction to Ordinary Differential Equations, Prentice-Hall of India.

3. G. F. Simmons, Differential Equations with Applications and Historical Notes, Second Edition, Tata McGraw Hill, India

4. Hurewicz W., Lectures on ordinary differential equations, M.I.T. Press.

5. Morris W. Hirsch and Stephen Smale, Differential Equations, Dynamical Systems, Linear Algebra, Elsevier.

PSMT105/PAMT105 DISCRETE MATHEMATICS

Course Outcomes

1. Students will solve Linear Diophantine equations, cubic equation by Cardanos Method, Quadratic Congruence equation. Students will learn the multiplicativity of function τ, σ and ϕ.

2. Students will be able to understand the proof of Erdos- Szekers theorem on monotone sub-sequences of a sequence with n^2+1 terms and the applicability of Forbidden Positions.

3. Stundent will learn the Fibonacci sequence, the Linear homogeneous recurrence relations with constant coefficient, Ordinary and Exponential generating Functions, exponential generating function for bell numbers, the applications of generating Functions to counting and use of generating functions for solving recurrence relations.

4. Stundents will be able to understand Polyas Theory of counting, Orbit stabilizer theorem, Burnside Lemma and its applications, Applications of Polya's Formula.

Unit I. Number theory (15 Lectures)
Divisibility, Linear Diophantine equations, Cardano's Method, Congruences, Quadratic residues, Arithmetic functions σ, τ, ϕ and their multiplicative property. Advanced counting: Types of occupancy problems, distribution of distinguishable and indistinguishable objects into distinguishable and indistinguishable boxes (with condition on distribution) Selections with Repetitions.

Unit II. Advanced counting (15 Lectures)
Stirling numbers of second and first kind. Pigeon-hole principle, generalized pigeonhole principle and its applications, Erdos-Szekers theorem on monotone subsequences, A theorem of Ramsey. Inclusion Exclusion Principle and its applications. Derangement. Permutations with Forbidden Positions, Restricted Positions and Rook Polynomials.

Unit III. Recurrence Relations (15 Lectures)
The Fibonacci sequence, Linear homogeneous and Non-homogeneous recurrence relations. Proof of the solution Linear homogeneous recurrence relations with constant coefficient in case of distinct roots and statement of the theorem giving a general solution (in case of repeated roots), Iteration and Induction. Ordinary generating Functions, Exponential Generating Functions, algebraic manipulations with power series, generating functions for counting combinations with and without repetitions, applications to counting, use of generating functions for solving homogeneous and non-homogeneous recurrence relations.

Unit IV. Polyas Theory of counting (15 Lectures)
Equivalence relations and orbits under a permutation group action. Orbit stabiliser theorem, Burnside Lemma and its applications, Cycle index, Polyas Formula, Applications of Polyas Formula.

Recommended Text Books

1. D. M. Burton, Introduction to Number Theory, McGraw-Hill.
2. Nadkarni and Telang, Introduction to Number Theory
3. V. Krishnamurthy:Combinatorics: Theory and applications, Affiliated East-West Press.
4. Richard A. Brualdi: Introductory Combinatorics, Pearson.
5. A. Tucker: Applied Combinatorics, John Wiley & Sons.
6. Norman L. Biggs: Discrete Mathematics, Oxford University Press.
7. Kenneth Rosen: Discrete Mathematics and its applications, Tata McGraw Hills.
8. Sharad S. Sane, Combinatorial Techniques, Hindustan Book Agency, 2013.

Semster-II
PSMT201 / PAMT201 ALGEBRA II

Course Outcomes

1. Students will learn Dihedral groups, Matrix groups, Automorphism group, Inner automorphisms, Structure theorem for finite abelain groups via examples.

2. Students will be able to understand group actions and orbit-stabilizer formula; Sylow theorems and applications to classification of groups of small order.

3. Students will be able to earn knowledge of prime avoidance theorem, Chinese remainder theorem, and specialized rings like Euclidean domains, principal ideal domains, unique factorization domains, their inclusions and counter examples.

Unit I. Groups and Group Homomorphisms (15 Lectures)
Review: Groups, subgroups, normal subgroups, center $Z(G)$ of a group. The kernel of a homomorphism is a normal subgroup. Cyclic groups. Lagrange's theorem. The product set $HK = \{hk/h \in H \ \& \ \in K\}$ of two subgroups of a group G: Examples of groups such as Permutation groups, Dihedral groups, Matrix groups, U_n-the group of units of $\mathbb{Z}_n$ (no questions be asked).

Quotient groups. First Isomorphism Theorem and the following two applications (reference: Algebra by Michael Artin)

1. Let $\mathbb{C}^*$ be the multiplicative group of non-zero complex numbers and $\mathbb{R} > 0$ be the multiplicative group of positive real numbers. Then the quotient group $\mathcal{C}^*/U$ is isomorphic to $\mathbb{R} > 0$:

2. The quotient group $GL_n(\mathbb{R})/SLn(\mathbb{R})$ is isomorphic to the multiplicative group of non-zero real numbers $\mathbb{R}^*$:

Second and third isomorphism theorems for groups, applications.

Product of groups. The group $\mathbb{Z}_m \times \mathbb{Z}_m$ is isomorphic to $\mathbb{Z}_{mn}$ if and only if $\gcd(m, n) = 1$. Internal direct product (A group G is an internal direct product of two normal subgroups H, K if $G = HK$ and every $g \in G$ can be written as $g = hk$ where $h \in H; k \in K$ in a unique way). If H, K are two finite subgroups of a group, then $|HK| = \frac{|H|\,|K|}{|H \cap K|}$. If H, K are two normal subgroups of a group G such that $H \cap K = \{e\}$ and $HK = G$, then G is internal direct product of H and K. If a group G is an internal direct product of two normal subgroups H and K then G is isomorphic to $H \times K$. (Reference: Algebra by Michael Artin) Inner automorphisms, Automorphisms of a group. If G is a group, then $A(G)$; the set of all automorphisms of G, is a group under composition. If G is a finite cyclic group of order r; then $A(G)$ is isomorphic to U_r; the groups of all units of $\mathbb{Z}_r$ under multiplication modulo r. For the infinite cyclic group Z; $A(Z)$ is isomorphic to $\mathbb{Z}_2$. Inner automorphisms of a group. (Reference: Topics in Algebra by I.N.Herstein). Structure theorem of Abelian groups (statement only) and applications (Reference: A first Course in Abstract Algebra by J. B. Fraleigh).

Unit II. Groups acting on sets and Sylow theorems
Center of a group, centralizer or normalizer $N(a)$ of an element $a \in G$; conjugacy class $C(a)$ of a in G: In finite group G; $|C(a)| = o(G)/o(N(a)$ and $o(G) = \sum o(G)/o(N(a)$ where the summation is over one element in each conjugacy class, applications such as:

1. If G is a group of order p^n where p is a prime number, then $Z(G) \neq \{e\}$:
2. Any group of order p^2; where p is a prime number, is Abelian. (Reference: Topics in Algebra by I. N. Herstein).

Groups acting on sets, Class equation, Cauchy's theorem: If p is a positive prime number and $p|o(G)$ where G is finite group, then G has an element of order p. (Reference: Topics in Algebra by I. N. Herstein). p-groups, Sylow theorems and applications:

1. There are exactly two isomorphism classes of groups of order 6:
2. Any group of order 15 is cyclic

(Reference for Sylow's theorems and applications: Algebra by Michael Artin).

Unit III. Rings and Fields (15 lectures)
Review: Rings (with unity), ideals, quotient rings, prime ideals, maximal ideals, ring homomorphisms, characteristic of a ring, first and second Isomorphism theorems for rings, correspondence theorem for rings (If $f : R \to R'$ is a surjective ring homomorphism, then there is a $1-1$ correspondence between the ideals of R containing the ker f and the ideals of R'). Integral domains, construction of the quotient field of an integral domain. (no questions be asked).

For a commutative ring R with unity:

1. An ideal M of R is a maximal ideal if and only if the quotient ring R/M is a field.
2. An ideal N of R is a prime ideal if and only if the quotient ring R/M is an integral domain.
3. Every maximal ideal is a prime ideal.
4. Every proper ideal is contained in a maximal ideal.
5. If an ideal I is contained in union of prime ideals $P_i, P_2, \cdots, P_n$, then I is contained in some P_i.

6. If a prime ideal P contains an intersection of ideals $I_i, I_2, \cdots, I_n$, then P contains some ideal I_j.

Rings of fractions, inverse and direct imges of ideals, Comaximal ideals, Chinese Remainder Theorem in rings and its applications to congruences.

Definition of field, characteristic of a field, sub field of a field. A field contains a sub field isomorphic to $\mathbb{Z}_p$ or $\mathbb{Q}$:

Polynomial ring $F[X]$ over a field, irreducible polynomials over a field. Prime ideals, and maximal ideals of a Polynomial ring $F[X]$ over a field F: A non-constant polynomial $p(X)$ is irreducible in a polynomial ring $F[X]$ over a field F if and only if the ideal $(p(X))$ is a maximal ideal of $F[X]$: Unique Factorization Theorem for polynomials over a field (statement only).

Unit IV. Divisibility in integral domains (15 lectures)
Prime elements, irreducible elements, Unique Factorization Domains, Principle Ideal Domains, Gauss's lemma, $Z[X]$ is a UFD, irreducibility criterion, Eisenstein's criterion, Euclidean domains. $Z[\sqrt{-5}]$ is not a UFD.
Reference for Unit IV: Michael Artin: Algebra, Prentice-Hall India.

Recommended Text Books

1. Michael Artin: Algebra, Prentice-Hall India.
2. I.N. Herstein: Topics in Algebra, Wiley-India.
3. R.B.J.T. Allenby: Rings, fields and Groups, An Introduction to Abstract Algebra, Elsevier (Indian edition).
4. J. B. Fraleigh, A first Course in Abstract Algebra, Narosa.
5. David Dummit, Richard Foot: Abstract Algebra, Wiley-India.

PSMT202 / PAMT202 TOPOLOGY

Course Outcomes

1. To understand a formation of new spaces from old one using product, box and quotient topology.
2. This course create a building block for analysis as well as algebraic geometry.
3. Students will understand extension theorems (e.g. Tietze extension theorem) which is useful in Functional Analysis.
4. This course covers the Tychonoff theorem which is a mile-stone of this subject.

Unit I.Topology and Topological spaces (15 Lectures)
Topological spaces, basis, topology generated by basis, sub-basis, order topology, product topology, subspace topology, closed sets, limit points, closure, interior, continuous functions, homeomorphism, box topology, comparison of the box and product topologies, T_0, T_1 spaces, For a T_1 space X, $x \in A \subset X$ is limit point of A if and only if every neighbourhood of x contains infinitely many points if A. Hausdorff space.

Unit II.Connected topological spaces (15 Lectures)
Quotient spaces. Connected topological spaces, separation of a topological space, continuity and connected-ness, path-connected topological spaces, topologist's sine curve, the order square I_0^2 is connected but not path connected. For $\mathbb{R}$ equipped with usual topology, the infinte cartesian product $\mathbb{R}^\omega$ in the product topology is connected but in box topology it is not. Connected components of a topological space, Path components of a topological space. Countability Axioms, first and second countable spaces, Separable spaces, Lindeloff spaces.

Unit III. Compact topological spaces (15 Lectures)
Compact spaces, continuity and compactness, tube lemma, finite product of compact topological spaces is compact. Finite intersection property, the Lebesgue number lemma, uniform continuity theorem, compact Hausdorff space with no isolated points is uncountable. Limit point compact spaces, local compactness, one point compactification.

Unit IV. Metrizable spaces and Tychonoff theorem (15 Lectures)
Metrizable spaces, separation axioms (regular and normal spaces). Every metrizable space is normal. A compact T_2 space is a normal space. Urysohn lemma, Urysohn metrization theorem, Tietze extension theorem. Tychonoff theorem,

Recommended Text Books

1. J. F. Munkres: Topology, Pearson; 2 edition (January 7, 2000).

2. G. F. Simmons: INtroduction to Topology and Modern Analysis, Tata McGraw Hill, 2004.

PSMT 203/PAMT 203: ANALYSIS II

Course Outcome:

1. In this course students are expected to understand the basic concepts of measure on an arbitrary measure space X as well as on $\mathbb{R}^n$.

2. They are also expected to study Lebesgue outer measure of sets and measurable sets, measurable functions.

3. Students will be able to understand the concepts of integrals of measurable functions in an arbitrary measure space $(X, \mathcal{A}, \mu)$. Lebesgue integration of complex valued functions and basic concepts of signed measures.

Unit-I: Measures and Measurable Sets (15 Lectures)
Additive set functions, σ-algebra countable additivity, Outer measure, constructing measures, μ^* measurable sets (Definitions due to Carathéodory), μ^* measurable subsets of X forms a σ algebra, measure space $(X, \sum, \mu)$. Lebesgue outer measure in $\mathbb{R}^d$, properties of exterior measure, monotonicity property and countable sub-additivity property of Lebesgue measure, translation invariance of exterior measure, example of set of measure zero. Measurable sets and Lebesgue measure, properties of measurable sets. Existence of a subset of $\mathbb{R}$ which is not Lebesgue measurable.
[Reference for unit I: 1. Andrew Browder, Mathematical Analysis, An Introduction, Springer Undergraduate Texts in Mathematics.
2. Elias M. Stein and Rami Shakarchi, Real Analysis, Measure Theory, Integration and Hilbert Spaces, New Age International Limited, India]

Unit-II: Measurable functions and their Integration (15 Lectures)
Measurable functions on $(X, \sum, \mu)$, simple functions, properties of measurable functions. If $f \geq 0$ is a measurable function, then there exists a monotone increasing sequence (s_n) of non-negative simple measurable functions converging to point wise to the function f. Egorov's theorem, Lusin's theorem. Integral of nonnegative simple measurable functions defined on the measure space $(X, \sum, \mu)$ and their properties. Integral of a non-negative measurable function.
[Reference for unit II: 1. Andrew Browder, Mathematical Analysis, An Introduction, Springer Undergraduate Texts in Mathematics.
2. Elias M. Stein and Rami Shakarchi, Real Analysis, Measure Theory, Integration and Hilbert Spaces, New Age International Limited, India]

Unit-III: Convergence Theorems on Measure space (15 Lectures)
Monotone convergence theorem. If $f \geq 0$ and $g \geq 0$ are measurable functions, then $\int (f + g) d\mu = \int f d\mu + \int g d\mu$, Fatou's lemma, summable functions, vector space of summable functions, Lebesgue's dominated convergence theorem. Lebesgue integral of bounded functions over a set of finite measure, Bounded convergence theorem. Lebesgue and

Riemann integrals: A bounded real valued function on [a, b] is Riemann integrable if and only if it is continuous at almost every point of [a, b]; in this case, its Riemann integral and Lebesgue integral coincide.
[Reference for unit III: 1. Andrew Browder, Mathematical Analysis, An Introduction, Springer Undergraduate Texts in Mathematics.
2. Royden H. L. Real Analysis, PHI]

Unit-IV: Space of Integrable functions (15 Lectures)
Borel set, Borel algebra of $\mathbb{R}^d$. Any closed subset and any open subset of $\mathbb{R}^d$ is Lebesgue measurable. Every Borel set in $\mathbb{R}^d$ is Lebesgue measurable. For any bounded Lebesgue measurable subset E of $\mathbb{R}^d$, given any $\epsilon > 0$ there exist a compact set K and open set U in $\mathbb{R}^d$ such that $K \subseteq E \subseteq U$ and $m(U \setminus K) < \epsilon$. For any Lebesgue measurable subset E of $\mathbb{R}^d$, there exist Borel sets F, G in $\mathbb{R}^d$ such that $F \subseteq E \subseteq G$ and $m(E \setminus F) = 0 = m(G \setminus E)$. Signed Measures, positive set, negative set and null set. Hahn decomposition theorem. Complex valued Lebesgue measurable functions on $\mathbb{R}^d$. Lebesgue integral of complex valued measurable functions, Approximation of Lebesgue integrable functions by continuous functions. The space $L^1(\mu)$ of integrable functions, properties of L^1 integrable functions, Riesz-Fischer theorem.
[Reference for unit IV: 1. Elias M. Stein and Rami Shakarchi, Real Analysis, Measure Theory, Integration and Hilbert Spaces, New Age International Limited, India
2. Royden H. L. Real Analysis, PHI
3. Andrew Browder, Mathematical Analysis, An Introduction, Springer Undergraduate Texts in Mathematics.]

Recommended Text Books

1. Andrew Browder, Mathematical Analysis, An Introduction, Springer Undergraduate Texts in Mathematics.

2. Elias M. Stein and Rami Shakarchi, Real Analysis, Measure Theory, Integration and Hilbert Spaces, New Age International Limited, India

3. Royden H. L. Real Analysis, PHI.

4. Terence Tao, Analysis II, Hindustan Book Agency (Second Edition).

PSMT 204 / PAMT 204 : PARTIAL DIFFERENTIAL EQUATIONS

Course Outcomes

1. Students are expected to understand the basic concepts and method of finding the solution of first and second order Partial Differential Equations (PDEs).
2. Students will be able to know the classification of second order PDEs, singularity and fundamental solution.
3. Students will be able to know the role of Green's function in the solution of Partial Differential Equations.
4. Through this course students will understand existence and uniqueness of solutions to Diffusion and Parabolic equations.

Unit-I: First Order Partial Differential Equations (15 Lectures)
First order partial differential equations in two independent variables, Semilinear and Quasilinear equations in two independent variables, method of characteristics, the Characteristics Cauchy Problem, General solutions.
Non-linear equations in two independent variables: Monge Strip and Charpit Equations, Solution of Cauchy problem, Determination of Complete integral, solution of Cauchy problem,
[Reference Unit-I (1.1,1.2, 1.3, 1.4)of Phoolan Prasad & Renuka Ravindran, Partial Differential Equations, Wiley Eastern Limited, India]

Unit-II: Second Order Partial Differential Equations (15 Lectures)
Classifications of second order partial differential equations in two and more than two independent variables, method of reduction to normal form, the Cauchy problem. Potential theory and elliptic differential equations, boundary value problems and Cauchy problem, Poisson's theorem, the mean value and the Maximum-Minimum properties
[Reference Unit-II (2.1, 2.2) of Phoolan Prasad & Renuka Ravindran, Partial Differential Equations, Wiley Eastern Limited, India]

Unit-III: Green's Functions and Integral Representations (15 Lectures)
Singularity functions and the fundamental solution, Green functions, Greens identities, Green's function for m-dimensions sphere of radius R, Green's functions Dirichlet problem in the plane, Neumann's function in the plane.
[Reference Unit-II (2.2.2):1. Phoolan Prasad & Renuka Ravindran, Partial Differential Equations, Wiley Eastern Limited, India and 2. Unit VIII of Yehuda Pinchover and Jacob Rubistein, An Introduction to Partial Differential Equations, Cambridge University Press]

Unit-IV: The Diffusion Equation & Parabolic Differential Equations (15 Lectures)

Existence and Uniqueness theorem for initial value problem in an infinite domain, semi-infinite domain, one dimensional Heat equation, maximum and minimum principle for the heat equation and for some parabolic equations, one dimensional wave equation, boundary value problem for the one dimensional heat and wave equations, method of separation of variables.

[Reference Unit-II (2.3.1, 2.3.2, 2.3.3, 2.3.4, 2.4.1 2.4.8)of Phoolan Prasad & Renuka Ravindran, Partial Differential Equations, Wiley Eastern Limited, India]

Recommended Text Books

1. Phoolan Prasad & Renuka Ravindran, Partial Differential Equations, Wiley Eastern Limited, India.

2. Yehuda Pinchover and Jacob Rubistein, An Introduction to Partial Differential Equations, Cambridge University Press.

3. T.Amaranath, An Elemetary Course in Partial Differential Equations, Narosa.

4. F. John, Partial Differential Equations, Springer publications.

5. G.B. Folland, Introduction to partial differential equations, Prentice Hall.

PSMT205/PAMT205 PROBABILITY THEORY

Course Outcomes

1. Students will understand the concept of Modelling Random Experiments, Classical probability spaces, -fields generated by a family of sets, -field of Borel sets, Limit superior and limit inferior for a sequence of events.

2. Students will be able to know about probability measure, Continuity of probabilities, First Borel-Cantelli lemma, Discussion of Lebesgue measure on -field of Borel subsets of assuming its existence, Discussion of Lebesgue integral for non-negative Borel functions assuming its construction.

3. Students will be able to earn knowledge of discrete and absolutely continuous probability measures, conditional probability, total probability formula, Bayes formula.

4. Students will learn distribution of a random variable, distribution function of a random variable, Bernoulli, Binomial, Poisson and Normal distributions,

5. Students will be able to understand Chebyshev inequality, Weak law of large numbers, Convergence of random variables, Kolmogorov strong law of large numbers, Central limit theorem and Application of Probability Theory.

Unit I. Probability basics (15 Lectures)
Modelling Random Experiments: Introduction to probability, probability space, events. Classical probability spaces: uniform probability measure, fields, finite fields, finitely additive probability, Inclusion-exclusion principle, σ-fields, σ-fields generated by a family of sets, σ-field of Borel sets, Limit superior and limit inferior for a sequence of events.

Unit II. Probability measure (15 Lectures)
Probability measure, Continuity of probabilities, First Borel-Cantelli lemma, Discussion of Lebesgue measure on σ-field of Borel subsets of assuming its existence, Discussion of Lebesgue integral for non-negative Borel functions assuming its construction. Discrete and absolutely continuous probability measures, conditional probability, total probability formula, Bayes formula, Independent events.

Unit III. Random variables (15 Lectures)
Random variables, simple random variables, discrete and absolutely continuous random variables, distribution of a random variable, distribution function of a random variable, Bernoulli, Binomial, Poisson and Normal distributions, Independent random variables, Expectation and variance of random variables both discrete and absolutely continuous.

Unit IV. Limit Theorems (15 Lectures)
Conditional expectations and their properties, characteristic functions, examples, Higher moments examples, Chebyshev inequality, Weak law of large numbers, Convergence of random variables, Kolmogorov strong law of large numbers (statement only), Central limit theorem (statement only).

Recommended Text Books

1. M. Capinski, Tomasz Zastawniak: Probability Through Problems.
2. J. F. Rosenthal: A First Look at Rigorous Probability Theory, World Scientic.
3. Kai Lai Chung, Farid AitSahlia: Elementary Probability Theory, Springer Verlag.
4. Ross, Sheldon M. A first course in probability(8th Ed), Pearson.

Scheme of Examination
The scheme of examination for the syllabus of Semesters I & II of M.A./M.Sc. Programme (CBCS) in the subject of Mathematics will be as follows.
Scheme of Evaluation R8435 for M. Sc /M. A.

1. A) 80: 20 for distance education (external evaluation of 80 marks and internal evaluation of 20 marks) under the choice based credit system (CBCS).
2. B) 60:40 for university affiliated PG centers (external evaluation of 60 marks and internal evaluation of 40 marks).
3. C) 100 percent internal evaluation scheme for University department of mathematics (One mid semester test of 30 marks, 05 marks for attendance, 05 marks for active participation and one end semester test of 60 marks, both tests will be conducted by the department and answer book will be shown to the students).

Duration:- Examination shall be of 2 and 1/2 Hours duration.
Theory Question Paper Pattern for Scheme B and Scheme C :-

1. There shall be five questions each of 12 marks.
2. On each unit there will be one question and the fifth one will be based on entire syllabus.
3. All questions shall be compulsory with internal choice within each question.
4. Each question may be subdivided into sub-questions a, b, c, and the allocation of marks depend on the weight-age of the topic.
5. Each question will be of maximum 18 marks when marks of all the sub-questions are added (including the options) in that question.
6. For scheme A: 60 marks will be converted into 80 marks.

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